**33.59** **In a hydronic system, $50gpm$ of water is supplied to two parallel branches from a common supply header and returned to a common return header. Branch A contains $80$ linear feet of piping and branch B contains $220$ linear feet of piping. Both branches contain a single water coil with a pressure drop of $15psi$ and a control valve with a pressure drop of $5psi$. All piping is the same diameter and the friction factor is constant throughout the system. What is the volume flow rate through branch A?**

A. $20.4gpm$

B. $29.6gpm$

C. $31.2gpm$

D. $36.7gpm$

In a parallel piping system, the pressure drop through each branch is equal. Write the Darcy-Weisbach Equation for both branches and set them equal.

$$h_{f,A} = h_{f,B}$$

$$\frac{fL_A v_A^2}{2Dg} = \frac{fL_B v_B^2}{2Dg}$$

Since the friction factor and diameter are the same throughout the system, all constant terms may be cancelled on both sides, leaving only length and velocity. Rearrange to obtain an expression for the ratio of velocities.

$$L_A v_A^2 = L_B v_B^2$$

$$\frac{v_A}{v_B} = \sqrt{\frac{L_B}{L_A}}$$

Since the diameter of the pipe is the same for both branches, the areas will also be the same. Using the relation $Q = vA$, the ratio of velocities may be replaced by the ratio of volume flow rates.

$$\frac{Q_A}{Q_B} = \frac{v_A A}{v_B A} = \frac{v_A}{v_B}$$

$$\frac{Q_A}{Q_B} = \sqrt{\frac{L_B}{L_A}}$$

In addition to the physical length of piping, it is necessary to add the equivalent length for the pressure drop associated with the coil and valve in each branch. Multiply by the rule of thumb conversion factor of $2.31\frac{ft}{psi}$, then add this amount to each branch's physical length as given.

$$\Delta p = 15psi + 5psi = 20psi\left(2.31\frac{ft}{psi}\right) = 46.2ft$$

$$L_A = 80ft + 46.2ft = 120.2ft$$

$$L_B = 220ft + 46.2ft = 260.2ft$$

Substitute into the previous expression to quantify $\frac{Q_A}{Q_B}$. Solve for $Q_B$ for use in the subsequent step.

$$\frac{Q_A}{Q_B} = \sqrt{\frac{L_B}{L_A}} = \sqrt{\frac{260.2ft}{120.2ft}} = 1.45$$

$$Q_A = 1.45Q_B$$

$$Q_B = 0.689Q_A$$

The sum of the volume flow rates through the two branches must equal the total, $50gpm$. Substitite for $Q_B$ and solve for $Q_A$.

$$Q_B + Q_A = 50gpm$$

$$0.689Q_A + Q_A = 50gpm$$

$$1.689Q_A = 50gpm$$

$$Q_A = 29.6gpm$$

**Answer B**